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Design of a Nuclear Reactor Controller Using a Model Predictive Control Method

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A model predictive controller is designed to control thermal power in a nuclear reactor. The basic concept of the model predictive control is to solve an optimization problem for finite future time steps at current time, to implement only the first optimal control input among the solved control inputs, and to repeat the procedure at each subsequent instant. A controller design model used for designing the model predictive controller is estimated every time step by applying a recursive parameter estimation algorithm. A 3-dimensional nuclear reactor analysis code, MASTER that was developed by Korea Atomic Energy Research Institute (KAERI), was used to verify the proposed controller for a nuclear reactor. It was known that the nuclear power controlled by the proposed controller well tracks the desired power level and the desired axial power distribution.

Key Words : Nuclear Reactor Control, Model Predictive Control, Load-following Operation, Nuclear Power Level Control, Axial Shape Index (ASI) Control, Recursive Parameter Estimation

1. Introduction

Nuclear reactor power and temperature should be properly controlled to establish good operation performance and also to maximize the thermal efficiency of nuclear power plants. But nuclear power plants are highly complex, nonlinear, time-varying, and constrained systems. At present, most nuclear power plants are operated at a base load, at it were, 100 percent rated power and don't depend largely on power tracking control except for startup and some problem occurrences. However, if the nuclear power plant electricity generation exceeds 60 percent of total electricity generation, it is said that nuclear power plants should have load-following operation capability, which means that the nuclear electricity generation should change according to the outside load. Also, if power exchange market is established well, it is considered that nuclear power plants should have load-following operation capability even at lower percent.

Until now, the fully automatic power tracking control of nuclear reactors has not been accepted in Korea due to the safety concerns of imprecise knowledge about the time-varying parameters,

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nonlinearity, and modeling uncertainty. However, rapid and smooth power maneuvering has its benefits in view of the economical and safe operation of reactors and the importance of load-following strategy.

A digital processor offers flexibility because the control function can be altered by software and this facilitates provisions of sophisticated control. Also, instrumentation and control (I&C) technology has been improved rapidly. In spite of these positive aspects of using a digital controller, for many reasons modern control systems have not been incorporated extensively in nuclear power plants. However, problems created by growing obsolescence of existing technology have stimulated interest in upgrading these systems (EPRI, 1992).

The model predictive control methodology has received much attention as a powerful tool for the control of industrial process systems (Kwon and Pearson, 1977 ; Richalet et al., 1978 ; Garcia et al., 1989 ; Clarke and Scattolini, 1991 ; Kothare et al., 1996 ; Lee et al., 1997 ; Lee et al., 1998). The basic concept of the model predictive control is to solve an optimization problem for a finite future at current time and to implement the first optimal control input as the current control input. That is, at the present time the behavior of the process over a horizon is considered and the process output is predicted by using a mathematical design model. The moves of the manipulated variables are selected such that the predicted output has certain desirable characteristics. However, only the first computed change in the manipulated variable is implemented and at each subsequent instant, the procedure is repeated. This method has many advantages over the conventional infinite horizon control because it is possible to handle input and state (or output) constraints in a systematic manner during the design and implementation of the control. In particular, it is a suitable control strategy for nonlinear time varying systems because of the model predictive concept. And recently, the problem of controlling uncertain dynamical systems has been of considerable interest to control engineers. Therefore, in this work the model predictive control method is applied to control the nuclear reactor core.

2. Conventional Nuclear Reactor Control System

Existing pressurized water reactors (PWRs) have two kinds of control mechanisms such as control rods and chemical shim to change the degree of reactor criticality for the purpose of raising or lowering the power level, and to keep a nuclear reactor critical by compensating for the changes in the properties of the system that takes place over its lifetime. In many power reactors, the control rod drives are interconnected electrically so that several control rods move simultaneously in response to a signal from the reactor operator. For example, for Korea Standard Nuclear Power Plants (KSNPs) a total of 73 control rods are divided into 2 shutdown control rod banks, 5 regulating control rod banks, and 2 part-strength control rod banks which are independently moving groups. As far as a reactor operator is concerned, KSNPs have only nine independent control rods. Among these ones, 5 regulating control rod banks and 2 part-strength control rod banks are used to control the reactor power level and the axial power distribution, respectively. Most PWRs can be controlled in part by varying the concentration of boric acid (H_3BO_3) in the coolant because boron absorbs neutrons well. Such chemical shim control cannot be used alone to control a nuclear reactor since the process of changing the boric acid concentration, although done remotely and automatically at the behest of the reactor operator, cannot be made to respond as quickly as control rods to sudden control requirements. Therefore, chemical shim is always used in conjunction with and as a supplement to mechanical control rods. When these two types of control mechanisms are present in a reactor, the control rods provide reactivity control for fast shutdown and for compensating for reactivity changes due to temperature changes that accompany changes in power. The chemical shim is used to keep the reactor critical during xenon transients and to compensate for depletion of fuel and buildup of fission product over the life of the reactor core.

The conventional reactor regulating system controls the average temperature of the reactor core according to the reference temperature that is proportional to the turbine load in order to maximize the plant thermal efficiency. The conventional reactor regulating system is described in Fig. 1. The conventional control method generates a control signal using a temperature deviation channel (the difference between the reference average coolant temperature and the average coolant temperature) and a power mismatch channel (difference between the turbine load and the nuclear power). As it were, the conventional controller generates the insertion or withdrawal speed of the reactor control rods using the error signals obtained by compensating and filtering these two channels. Finally, the control rod drive mechanism control system moves the control rod assembly groups according to the received signals. This conventional method has its own advantages of easy implementation and well-proven technology. However, in order to optimize the reactor power control performance, techniques for the optimal power control of nuclear reactors have been studied extensively in the past two decades (Lin et al., 1989 ; Niar and Gopal, 1987 ; Park and Cho, 1993).

The explanations described above are related to controlling the total reactor power. Axial power distribution unbalance which is usually caused by the frequent movement of control rod assemblies for the daily load-following operation induces xenon oscillation in a reactor core because the neutron absorption cross-section of xenon is extremely large and its effects in a nuclear reactor are delayed by the iodine precursor. The physical explanation of xenon-induced spatial power oscillations is well described in literature (Duderstadt and Hamilton, 1976). Therefore, if daily load-following operation is performed, it is necessary to pay attention to controlling the axial

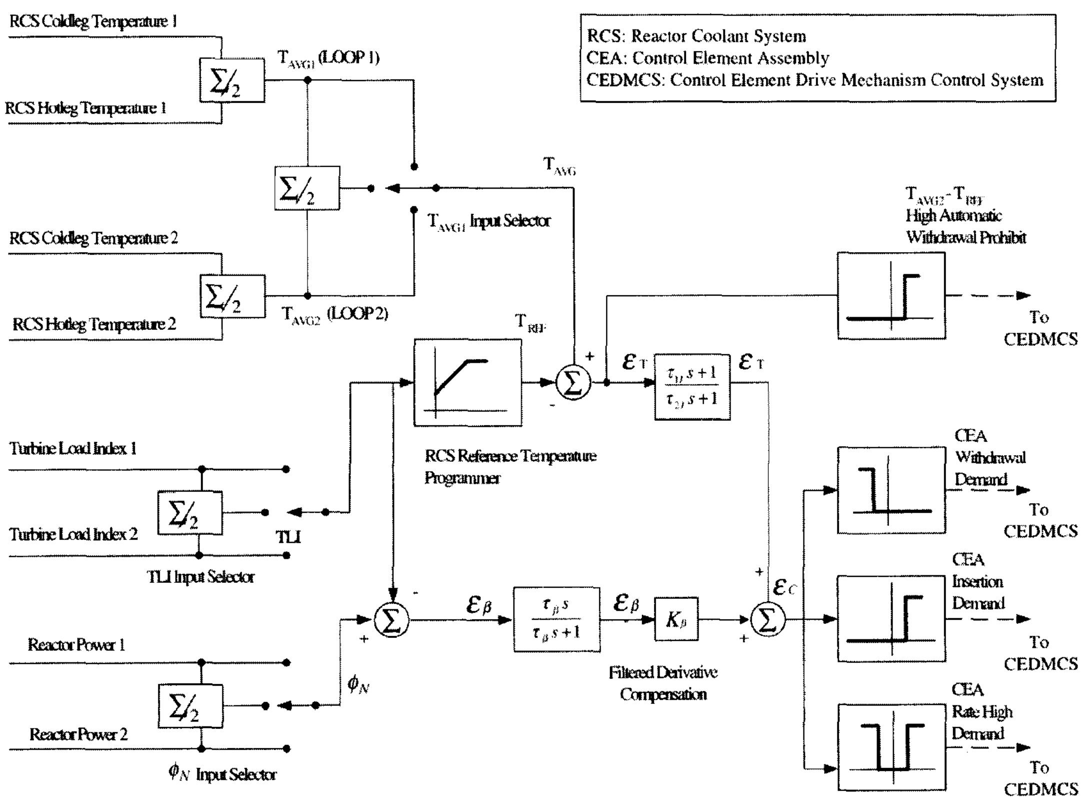


Fig. 1 Conventional reactor regulating system

power distribution in a nuclear reactor in order to uniformly burn nuclear fuels. Because the reactor power distribution control has been one of the most challenging control problems in the nuclear field, there have been extensive research activities in this area, especially using conventional optimal control methods (Cho and Grossman, 1983; Winokur and Tepper, 1984; Yoon and No, 1985; Gondal and Axford, 1986; Park and Cho, 1992). The load-following operation control called as Mode-K had been developed for KSNPs although it is not being applied now (Choi et al., 1992; Kim et al., 1999).

The Mode-K was designed to automatically control the axial power distribution as well as the core power, i.e., the core average temperature by mainly using the control element assemblies (CEAs), with operator control of the concentration of boron dissolved in coolant during the load-following operation. CEAs of KSNPs consist of 5 regulating control rod banks, 2 part-strength control rod banks, and 2 shutdown control rod banks. Among these ones, 5 regulating control rod banks and 2 part-strength control rod banks are used to control the reactor power level and the axial power distribution. The control rod bank selection logic of Mode-K depends on the magnitude of the axial shape index (ASI) deviation (ΔASI$\equiv$ASI-target ASI) that is categorized by 5 stage flags. Here ASI is defined as $\frac{P_B - P_T}{P_B + P_T}$ where P_B is the bottom half power of a reactor core and P_T is its top half power. The stage flag varies as the ASI deviation changes as shown in Fig. 2. The ARS+ (ASI Restoring Stage) and ARS- stage flags mean bottom-shifted and top-shifted power distributions, respectively. The bottom-shifted power distribution means $P_B > P_T$. For ARS stage flags, Mode-K tries to select CEA banks to restore the ASI. FOS+ (Fixed Overlap Stage) denotes slightly bottom-shifted power profile, while FOS- slightly top-shifted profile. During FOS, the ASI deviation is considered as acceptable and thus all CEA banks are moved simultaneously in the fixed overlap mode to control the core reactivity. When the ASI mismatch is very small, the stage flag is ORS (Overlap Restoring Stage). The ORS stage flag indicates that the CEA movement should be done such that the reference overlap between CEA banks could be restored, regardless of the ASI change resulting from the CEA movement.

For a specific core condition, the Mode-K logic selects the optimal CEA bank, depending on the CEA direction (insertion or withdrawal) and the stage flag. The setpoints in Fig. 2 for the hysteresis of the stage flag are determined via numerical simulations to maximize the performance of Mode-K. Basically, the bank selection logic of Mode-K is based on simple, well-known physical phenomena: insertion of a CEA in the top half of the core suppresses the top power, while CEA insertion in the bottom half decreases the bottom power. On the contrary, withdrawal of

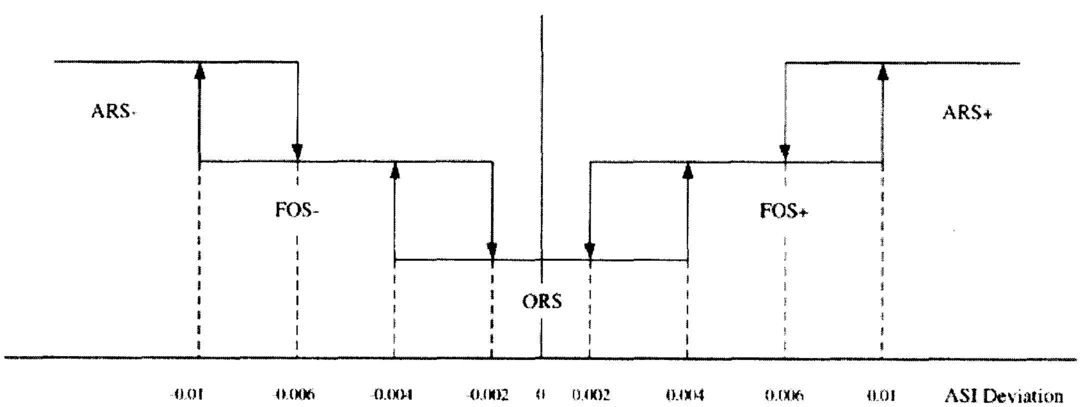


Fig. 2 Concept of Mode-K stage flag change

a CEA in the top half of the core results in top-shift of the power distribution relative to the initial state, and CEA withdrawal in the bottom half induces the bottom-shift of the power distribution.

3. Model Predictive Control Method

The conventional reactor control system uses conventional compensators and filters shown in Fig. 1 and also a heuristics method described in Fig. 2. In this work, a model predictive controller is designed to optimize the reactor control performance.

The model predictive control method is to solve an optimization problem for finite future time steps at current time and to implement the first optimal control input as the current control input. At the next time step, new values of the measured output are obtained, the control horizon is shifted forward by one step, and the same calculations are repeated. The purpose of taking new measurements at each time step is to compensate for unmeasured disturbances and model inaccuracy, both of which cause the measured system output to be different from the one predicted by the model. At every time instant, model predictive control requires the on-line solution of an optimization problem to compute optimal control inputs over a fixed number of future time instants, known as the time horizon.

The basic idea of model predictive control is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. Also, in order to achieve fast responses and prevent excessive control effort, the associated performance index for deriving an optimal control input is represented by the following quadratic function :

$$J=\frac{1}{2}\sum_{j=1}^{N}Q[\hat{y}(t+j\mid t)-w(t+j)]^2+\frac{1}{2}\sum_{j=1}^{M}R[\Delta u(t+j-1)]^2 \tag{1}$$

subject to constraints

$$\begin{cases} y(t+N+i)=w(t+N+i),\ i=1,\cdots,m \\ \Delta u(t+j-1)=0,\ j>M \end{cases}$$

where Q and R weight the output error $(\hat{y}-w)$ and the control input change between neighboring time steps at a certain future time interval, respectively, and w is a setpoint or reference sequence for the output signal. $\hat{y}(t+j\mid t)$ is an optimum j-step-ahead prediction of the system output based on data up to time t ; that is, the expected value of the output at time t if the past input and output and the future control sequence are known. N and M are called the prediction horizon and the control horizon, respectively. The prediction horizon represents the limit of the instant in which it is desired for the output to follow the reference sequence. In order to obtain control inputs, the predicted outputs have to be first calculated as a function of past values of inputs and outputs and of future control signals. The constraint, $\Delta u(t+j-1)=0$ for $j>M$, means that there is no variation in the control signals after a certain interval $M<N$, which is the control horizon concept. The constraint, $y(t+N+i)=w(t+N+i)$, $i=1, \cdots, m$, which makes the output follow the reference input over some range, guarantees the stability of the controller.

The process to be controlled is described by the following Controlled Auto-Regressive and Integrated Moving Average (CARIMA) model, which is widely used as a mathematical model of controller design methods :

$$A(q^{-1})y(t)=B(q^{-1})\Delta u(t-1)+\xi(t) \tag{2}$$

where y is an output, Δu is a control input change, ξ is a stochastic random noise sequence with zero mean value, and q^{-1} is the backward shift operator.

The process output at time $t+j$ can be predicted from the measurements of the output and input up to time step t. The optimal prediction is derived by solving a Diophantine equation, whose solution can be found by an efficient recursive algorithm. By taking the expectation operator and considering that $E\{\xi(t)\}=0$, the optimal j-step-ahead prediction of $\hat{y}(t+j\,|\,t)$ satisfies

$$\begin{aligned}\hat{y}(t+j\,|\,t)=&\bar{G}_j(q^{-1})\Delta u(t+j-1)\\&+\tilde{G}_j(q^{-1})\Delta u(t-1)\\&+F_j(q^{-1})y(t)\end{aligned} \tag{3}$$

where

$$1=E_j(q^{-1})A(q^{-1})+q^{-j}F_j(q^{-1}) \tag{4}$$

$$E_j(q^{-1})=e_{j,0}+e_{j,1}q^{-1}+\cdots+e_{j,j-1}q^{-(j-1)} \tag{5}$$

$$F_j(q^{-1})=f_{j,0}+f_{j,1}q^{-1}+f_{j,2}q^{-2}+\cdots+f_{j,nA}q^{-nA}$$

$$G_j(q^{-1})=E_j(q^{-1})B(q^{-1})$$

$$\hat{y}(t+j\,|\,t)=E\{y(t+j)\,|\,t\} \tag{6}$$

$$G_j(q^{-1})=\bar{G}_j(q^{-1})+q^{-j}\tilde{G}_j(q^{-1})$$

$$\text{with } \deg(\bar{G}_j(q^{-1}))<j$$

$y(t+j\,|\,t)$ denotes an estimated value of the output at time step $t+j$ based on all the data up to time step t. The output prediction can easily be extended to the nonzero mean noise case by adding a term $E_j(q^{-1})E\{\xi(t)\}$ to the output prediction $\hat{y}(t+j\,|\,t)$. The last two terms of the right hand side of Eq. (3) consist of past values of the process input and output variables and correspond to the response of the process if the control input signals are kept constant. On the other hand, the first term of the right hand side consists of future values of the control input signal and correspond to the response obtained when the initial conditions are zero, as it were, $y(t-j)=0$ and $\Delta u(t-j-1)=0$ for $j>0$. Equation (3) can be rewritten as

$$\hat{\mathbf{y}}=\bar{\mathbf{G}}\Delta\mathbf{u}+\mathbf{f} \tag{7}$$

where

$$\hat{\mathbf{y}}=[\hat{y}(t+1|t)\ \hat{y}(t+2|t)\ \cdots\ \hat{y}(t+j|t)\ \cdots\ \hat{y}(t+N|t)]^T$$

$$\Delta\mathbf{u}=[\Delta u(t)\ \Delta u(t+1)\ \cdots\ \Delta u(t+j)\ \cdots\ \Delta u(t+N-1)]^T$$

$$\mathbf{f}=[f_1\ f_2\ \cdots\ f_j\ \cdots\ f_N]^T$$

$$f_j=\tilde{G}_j(q^{-1})\Delta u(t-1)+F_j(q^{-1})y(t) \tag{8}$$

$$\bar{\mathbf{G}}=\begin{bmatrix} g_0 & 0 & \cdots & 0 & \cdots & 0\\ g_1 & g_0 & \cdots & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ g_{j-1} & g_{j-2} & \cdots & g_0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ g_{N-1} & g_{N-2} & \cdots & \cdots & \cdots & g_0 \end{bmatrix}$$

$$\bar{G}_j(q^{-1})=\sum_{i=0}^{j-1}g_iq^{-i}$$

If all initial conditions are zero, the response $\mathbf{f}$ is zero. If a unit step is applied to the first input at time t; that is, $\Delta\mathbf{u}=[1\ 0\ \cdots\ 0]^T$, the expected output sequence $[\hat{y}(t+1)\,\hat{y}(t+2)\ \cdots\ \hat{y}(t+N)]^T$ is equal to the first column of the matrix $\bar{\mathbf{G}}$. That is, the first column of the matrix $\bar{\mathbf{G}}$ can be calculated as the step response of the plant when a unit step is applied to the first control signal.

The computation of the control input involves the inversion of a $N\times N$ matrix $\bar{\mathbf{G}}$ that requires a substantial amount of computation. If the control signal is kept constant after the first M control moves (that is, $\Delta u(t+j-1)=0$ for $j>M$) due to the model predictive control concept, this leads to the inversion of an $M\times M$ matrix, which reduces the amount of computation. If so, the set of predictions affecting the objective function can be expressed as

$$\hat{\mathbf{y}}=\bar{\mathbf{G}}_s\Delta\mathbf{u}_s+\mathbf{f} \tag{9}$$

where

$$\bar{\mathbf{G}}_s=\begin{bmatrix} g_0 & 0 & \cdots & 0\\ g_1 & g_0 & \cdots & 0\\ & & \ddots & \vdots\\ \vdots & \vdots & & g_0\\ & & & \vdots\\ g_{N-1} & g_{N-2} & \cdots & g_{N-M} \end{bmatrix}$$

$$\Delta\mathbf{u}_s=[\Delta u(t)\ \Delta u(t+1)\ \cdots\ \Delta u(t+M-1)]^T$$

The following relationship for the constraint of the foregoing objective function can be derived from the foregoing equation :

$$\hat{\mathbf{y}}_f=\bar{\mathbf{G}}_{sf}\Delta\mathbf{u}_s+\mathbf{f}_f \tag{10}$$

where

$$\hat{\mathbf{y}}_f=[\hat{y}(t+N+1|t)\ \hat{y}(t+N+2|t)\ \cdots\ \hat{y}(t+N+m|t)]^T$$

$$\mathbf{f}_f=[f_{N+1}\ f_{N+2}\ \cdots\ f_{N+m}]^T$$

$$\overline{\mathbf{G}}_{sf}=\begin{bmatrix} g_N & g_{N-1} & \cdots & g_{N-M+1} \\ g_{N+1} & g_N & \cdots & g_{N-M+2} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N+m-1} & g_{N+m-2} & \cdots & g_{N-M+m} \end{bmatrix}$$

The objective function of Eq. (1) can be rewritten as the following matrix-vector form :

$$\begin{aligned} J&=\frac{1}{2}(\hat{\mathbf{y}}-\mathbf{w})^T\tilde{\mathbf{Q}}(\hat{\mathbf{y}}-\mathbf{w})+\frac{1}{2}\Delta\mathbf{u}_s^T\tilde{\mathbf{R}}\Delta\mathbf{u}_s \\ &=\frac{1}{2}(\overline{\mathbf{G}}_s\Delta\mathbf{u}_s+\mathbf{f}-\mathbf{w})^T\tilde{\mathbf{Q}}(\overline{\mathbf{G}}_s\Delta\mathbf{u}_s+\mathbf{f}-\mathbf{w}) \\ &\quad+\frac{1}{2}\Delta\mathbf{u}_s^T\tilde{\mathbf{R}}\Delta\mathbf{u}_s \end{aligned} \tag{11}$$

$$\text{subject to } \mathbf{w}_f=\overline{\mathbf{G}}_{sf}\Delta\mathbf{u}_s+\mathbf{f}_f \tag{12}$$

where

$$\mathbf{w}=[w(t+1|t)\ w(t+2|t)\ \cdots\ w(t+N|t)]^T$$

$$\mathbf{w}_f=[w(t+N+1|t)\ w(t+N+2|t)\ \cdots\ w(t+N+m|t)]^T$$

$\tilde{\mathbf{Q}}=diag(Q, \cdots, Q)$ is a diagonal matrix consisting of N diagonal elements, Q, and $\tilde{\mathbf{R}}=diag(R, \cdots, R)$ is a diagonal matrix consisting of M diagonal elements, R. Usually $\tilde{\mathbf{Q}}=\mathbf{I}_{N\times N}$ and $\tilde{\mathbf{R}}=\omega\times\mathbf{I}_{M\times M}$ are used and ω is called an input-weighting factor.

The optimal input can be obtained by the well-known Lagrange multiplier approach. To apply the Lagrange multiplier approach, the objective function is rewritten as

$$\begin{aligned} J'&=\frac{1}{2}(\overline{\mathbf{G}}_s\Delta\mathbf{u}_s+\mathbf{f}-\mathbf{w})^T\tilde{\mathbf{Q}}(\overline{\mathbf{G}}_s\Delta\mathbf{u}_s+\mathbf{f}-\mathbf{w}) \\ &\quad+\frac{1}{2}\Delta\mathbf{u}_s^T\tilde{\mathbf{R}}\Delta\mathbf{u}_s+\lambda^T(\overline{\mathbf{G}}_{sf}\Delta\mathbf{u}_s+\mathbf{f}_f-\mathbf{w}_f) \end{aligned} \tag{13}$$

By solving the foregoing objective function using the Lagrange multiplier method, the following equation is obtained :

$$\begin{aligned} \Delta\mathbf{u}_s=&(\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}\overline{\mathbf{G}}_s+\tilde{\mathbf{R}})^{-1}[\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}(\mathbf{w}-\mathbf{f}) \\ &+\overline{\mathbf{G}}_{sf}^T[\overline{\mathbf{G}}_{sf}(\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}\overline{\mathbf{G}}_s+\tilde{\mathbf{R}})^{-1}\overline{\mathbf{G}}_{sf}^T]^{-1} \\ &[\mathbf{w}_f-\mathbf{f}_f-\overline{\mathbf{G}}_{sf}(\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}\overline{\mathbf{G}}_s+\tilde{\mathbf{R}})^{-1}[\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}(\mathbf{w}-\mathbf{f})]]] \end{aligned} \tag{14}$$

Calculating the control input requires the inversion of matrices $(\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}\overline{\mathbf{G}}_s+\tilde{\mathbf{R}})$ and $\overline{\mathbf{G}}_{sf}(\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}\overline{\mathbf{G}}_s+\tilde{\mathbf{R}})^{-1}\overline{\mathbf{G}}_{sf}^T$. From the definition of matrix $\overline{\mathbf{G}}_{sf}$, it can be derived that the number of output constraint, m, cannot be bigger than the number of control signal variations, M ; that is, $m\leq M$. Another condition for invertibility must be satisfied ; $m\leq n+1$ since the coefficient g_i of the step response is a linear combination of the previous $n+1$ values (n is the system order). Therefore, the inversion of matrix $\overline{\mathbf{G}}_{sf}(\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}\overline{\mathbf{G}}_s+\tilde{\mathbf{R}})^{-1}\overline{\mathbf{G}}_{sf}^T$ requires inverting a matrix of which the dimension m is not usually bigger than three or four. Since only $\Delta u(t)$ is needed at time step t, only the first row of the matrices, $(\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}\overline{\mathbf{G}}_s+\tilde{\mathbf{R}})^{-1}\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}$ and $(\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}\overline{\mathbf{G}}_s+\tilde{\mathbf{R}})^{-1}\overline{\mathbf{G}}_{sf}^T[\overline{\mathbf{G}}_{sf}(\overline{\mathbf{G}}_s^T\tilde{\mathbf{Q}}\overline{\mathbf{G}}_s+\tilde{\mathbf{R}})^{-1}\overline{\mathbf{G}}_{sf}^T]^{-1}$, is required to be computed. Also, in order to obtain the control input from Eq. (14), it is necessary to calculate the matrices $\overline{\mathbf{G}}_s$, and $\overline{\mathbf{G}}_{sf}$, and the vectors $\mathbf{f}$ and $\mathbf{f}_f$. These matrices and vectors can be calculated recursively as follows :

$$E_{j+1}(q^{-1})=E_j(q^{-1})+p_jq^{-j} \text{ with } p_j=f_{j,0} \tag{15}$$

$$f_{j+1,i}=f_{j,i+1}-p_ja_{i+1} \text{ for } i=0, \cdots, \delta(F_{j+1}) \tag{16}$$

$$E_1=1 \tag{17}$$

$$F_1=q(1-A(q^{-1})) \tag{18}$$

$$\begin{aligned} &f_{j+1}=q(1-A(q^{-1}))f_j+B(q^{-1})\Delta u(t+j) \\ &\text{with } f_0=y(t) \text{ and } \Delta u(t+j)=0 \text{ for } j\geq 0 \end{aligned} \tag{19}$$

$$\begin{aligned} G_{j+1}(q^{-1})&=E_{j+1}(q^{-1})B(q^{-1}) \\ &=G_j(q^{-1})+f_{j,0}q^{-j}B(q^{-1}) \end{aligned} \tag{20}$$

4. Recursive Parameter Estimation

The process model is estimated recursively every time step to reflect time-varying conditions of the plant including fuel burnup, control rod movement and so on. Equation (3) can be expressed in the following inner product of the parameter vector $\hat{\boldsymbol{\theta}}(t)$ and the measurement vector $\boldsymbol{\varphi}(t)$:

$$\hat{y}(t+1) = F_1(q^{-1})y(t) + G_1(q^{-1})\Delta u(t) \\ = \hat{\theta}^T(t)\cdot\varphi(t) \tag{21}$$

where

$$\hat{\theta}^T(t) = [\hat{a}_1(t)\ \hat{a}_2(t)\ \cdots\ \hat{a}_{nA}(t)\ \hat{b}_0(t)\ \hat{b}_1(t)\ \cdots\ \hat{b}_{nB}(t)]$$

$$\varphi^T(t) = [-y(t)\ -y(t-1)\ \cdots\ -y(t-nA+1)\ \Delta u(t)\ \Delta u(t-1)\ \cdots\ \Delta u(t-nB)]$$

The parameter vector $\hat{\theta}(t)$ is estimated using a recursive least squares method as follows:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \Sigma(t)\varphi(t-1)[y(t) - \hat{\theta}^T(t-1)\cdot\varphi(t-1)] \tag{22}$$

$$\Sigma(t) = \frac{1}{\lambda(t)}\left[\Sigma(t-1) - \frac{\Sigma(t-1)\varphi(t-1)\varphi^T(t-1)\Sigma(t-1)}{\lambda(t) + \varphi^T(t-1)\Sigma(t-1)\varphi(t-1)}\right] \tag{23}$$

where the covariance matrix $\Sigma(0) > 0$ and $0 < \lambda(t) \leq 1$. The forgetting factor $\lambda(t)$ is calculated from the following equation:

$$\lambda(t) = \lambda_0\lambda(t-1) + (1-\lambda_0) \\ \text{with } \lambda_0 \leq 1 \text{ and } \lambda(0) \leq 1 \tag{24}$$

The parameters estimated by Eqs. (22) through (24) are used to design the model predictive controller.

5. Application to Nuclear Reactor Power Control

Figure 3 shows the schematic block diagram of the model predictive controller combined with a parameter estimation algorithm. In this work, the developed controller was applied to a 3-dimentional reactor model (MASTER code). MASTER (Multipurpose Analyzer for Static and Transient Effects of Reactor) developed by KAERI is a nuclear analysis and design code which can simulate the Pressurized Water Reactor (PWR) and Boiling Water Reactor (BWR) cores in 3-dimensional geometry. MASTER was designed to have a variety of capabilities such as static nuclear reactor core design, transient nuclear reactor core analysis and operation support. The MASTER code is written in FORTRAN and the proposed control algorithm in MATLAB (MathWorks, 1999). Visual C++ is in charge of the variable transfer between the MASTER code and the control algorithm.

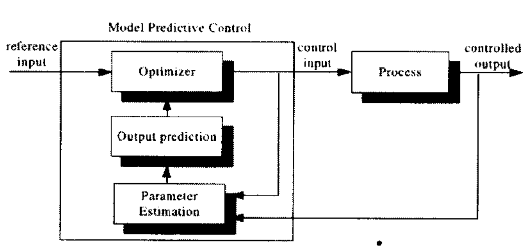


Fig. 3 Schematic block diagram of a model predictive controller combined with a parameter estimator

At first, a reactor power level controller using the model predictive control method was designed and applied to the Yonggwang unit 3 nuclear power plant (YGN-3) modeled by the MASTER code. The thermal power of YGN-3 are regulated by 5 regulating control rod banks and 2 part-strength control rod banks. Also, the thermal power is regulated by changing the concentration of boron that absorbs neutrons strongly and is dissolved in coolant. In this work, it is assumed that the power level is controlled by only the regulating control rod banks, R5, R4, R3, R2, and R1 and the boron concentration is not changed for hours when the depletion of nuclear fuel is

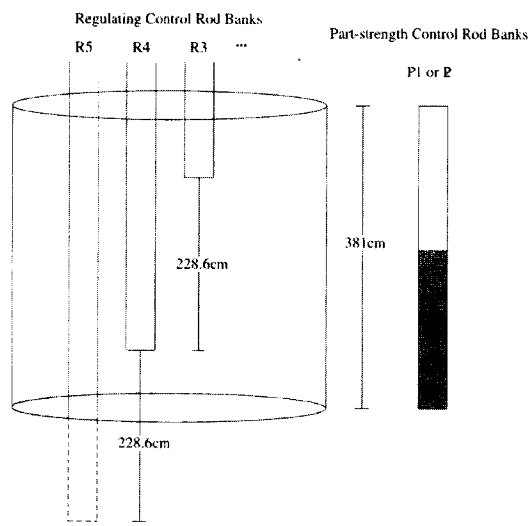
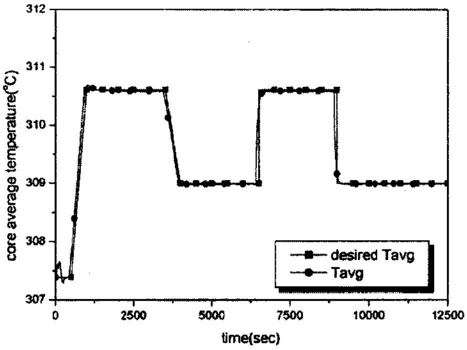
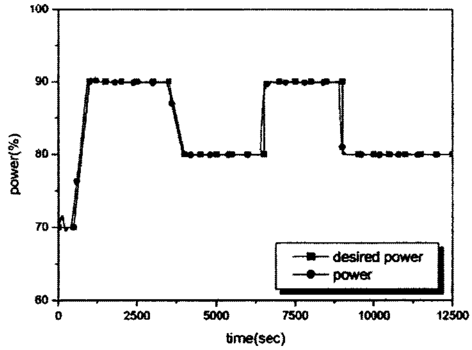


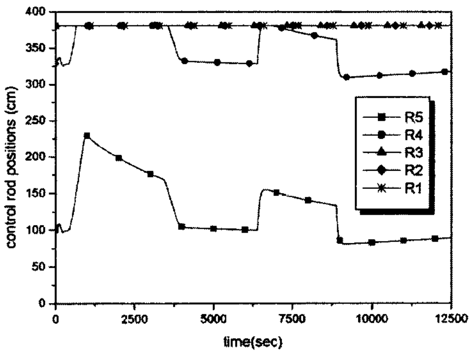
Fig. 4 Overlapped positions of the regulating control rod banks and only the bottom part of the part-strength control rod banks filled with the neutron absorber material



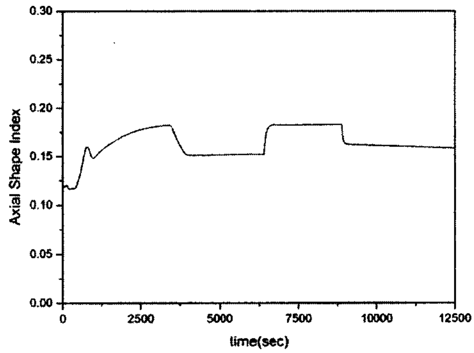
(a) Average coolant temperature



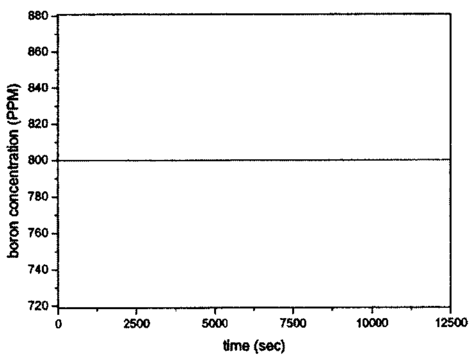
(b) Reactor power



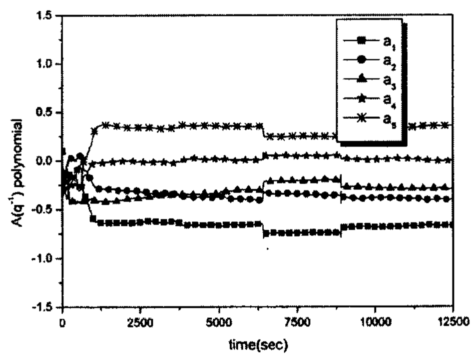
(c) Regulating control rod bank position



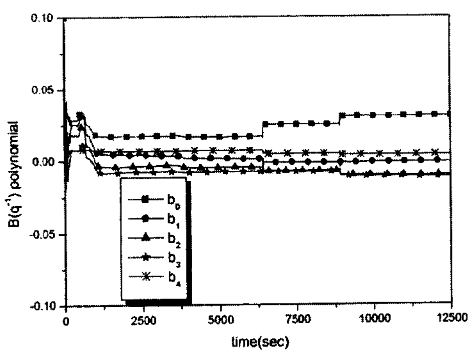
(d) Axial shape index (ASI)



(e) Boron concentration



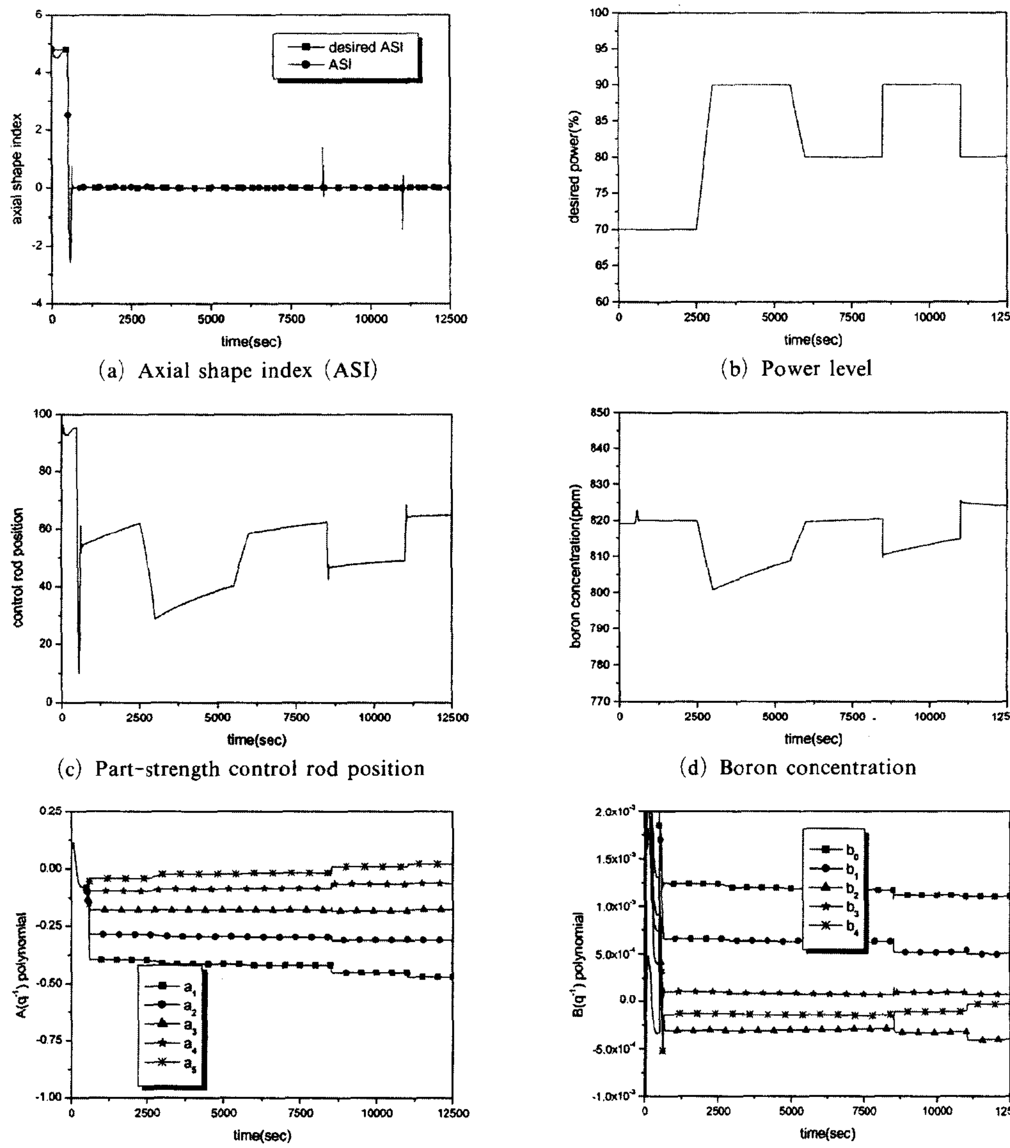
(f) $A(q^{-1})$ polynomial



(g) $B(q^{-1})$ polynomial

Fig. 5 Power level control

insignificant (refer to Fig. 5(e)). These regulating control banks are not totally independently moving. The control rod banks are inserted inside and withdrawn out of a reactor core by being overlapped between control rod banks (refer to Fig. 4). For example, in case they are inserted from the top position (381 cm), when the R5 control rod bank is inserted first and approaches 152.4 cm (=381−228.6 cm, 228.6 cm overlap) axial position, the R4 control rod bank goes into the reactor core together with the R5 control rod bank. As shown in Fig. 4, the positions of all regulating control rod banks can be described by the pseudo position of the regulating control rod bank R5 that is a control input. Figure 5 shows its simulation results. The desired power is 70% initially and changes by ramp and step. It is shown that the average coolant temperature



(a) Axial shape index (ASI)

(b) Power level

(c) Part-strength control rod position

(d) Boron concentration

(e) $A(q^{-1})$ polynomial

(f) $B(q^{-1})$ polynomial

Fig. 6 Axial shape index (ASI) control follows the desired average coolant temperature. Figure 5(d) shows the axial shape index (ASI). The ASI exceeds the desired ASI band. Therefore, the power level control should be taken into account together with the ASI control, which is the load-following control problem that resolves xenon-induced power oscillations due to control rod movement to follow the desired load.

Second, the reactor power distribution (or ASI) controller was designed and applied to YGN-3. Figure 6 shows its simulation results. The ASI is controlled by the part-strength control rod (control banks P1 or P2) of which the bottom half part contains neutron absorber material and the top half part does not (refer to Fig. 5). All the regulating control rod banks are withdrawn out of the reactor core. The desired power is 70% initially and changes by ramp and step (refer to Fig. 6(b)). The initial desired ASI is a value induced by P1 and P2 control rods insertion, which is initially positioned at 95.25 cm (refer to Fig. 6(c)), which means that the absorber material of the P control rod bank is positioned half and half between top and bottom parts. At this state, the ASI is 4.7%. The ASI is desired to maintain this level for initial 500 sec and then to be zero. Zero ASI means that the thermal powers of the top-half region and the bottom-half region of a reactor core are the same. Two peaks shown between about 7500 sec and 12500 sec of Fig. 6(a) are due to the step change of the desired power level. It is assumed that the power level is controlled completely by boron concentration, which does not reflect the real situation.

A constrained model predictive controller for the power level control was designed to compare with the prescribed generalized predictive controller. The constraints for the control input $u(t)$ and output $y(t)$ are as follows:

$$-914.4\ \text{cm} \le u(t) \le 381\ \text{cm}$$

$$-1.27 \times T \le \Delta u(t) \le 1.27 \times T$$

$$307.39^\circ\text{C} \le y(t) \le 323.98^\circ\text{C}$$

where

$$\Delta u(t) = u(t) - u(t-1)$$

The pseudo-position of the R5 regulating control bank that is a control input is larger than four overlap lengths (914.4 cm) below the bottom of reactor core and smaller than the top position of the reactor core (refer to Fig. 4). The maximum speed of the regulating control rods is 30 inches/min ($1.27\,T$ cm/sec where T is a sampling time). The reactor core average temperature that is a controlled output is between the temperature corresponding to hot zero reactor power and the temperature corresponding to 100% reactor power. The optimal control input could be obtained by solving the minimization objective of Eq. (1) combined with the above constraints through the semi-definite programming. Figure 7 shows the performance of the constrained model predictive controller whose responses are similar to those of Fig. 5. The performance difference between these two originates from a little difference concerning about control algorithm and system identification.

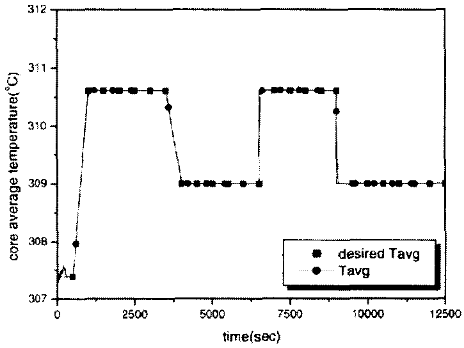
Also, a constrained model predictive controller for the ASI control was designed. The constraints for the control input and output of this controller are as follows:

$$0\ \text{cm} \le u(t) \le 381\ \text{cm}$$

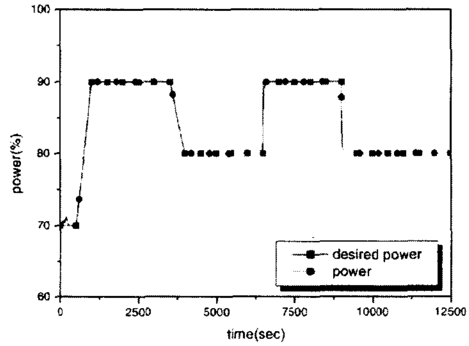
$$-1.27 \times T \le \Delta u(t) \le 1.27 \times T$$

$$-30\% \le y(t) \le 30\%$$

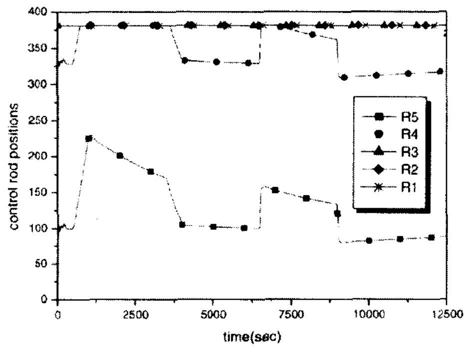
The two part-strength control banks, P1 and P2, move simultaneously in response to a signal. The position of the two part-strength control banks that is a control input is higher than the bottom of the reactor core and lower than the top (refer to Fig. 4). The controlled output of the axial shape index is between 30% top-skewed power distribution and 30% bottom-skewed one. Figure 8 shows the performance of the constrained model predictive controller whose responses are similar to those of Fig. 6, which is because the input and output of the non-constrained model predictive controller do not exceed the constraints.



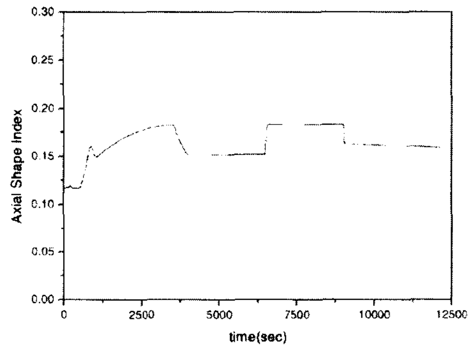
(a) Average coolant temperature



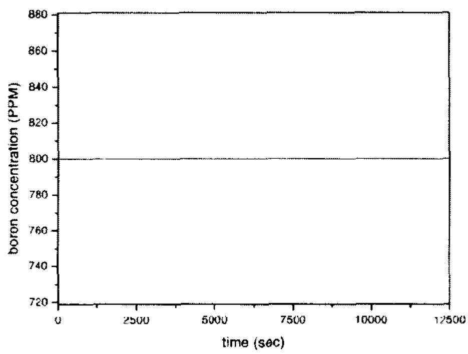
(b) Reactor power



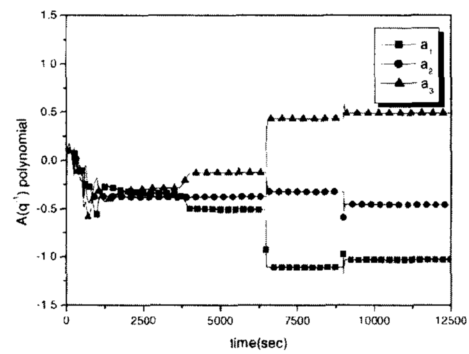
(c) Regulating control rod bank position



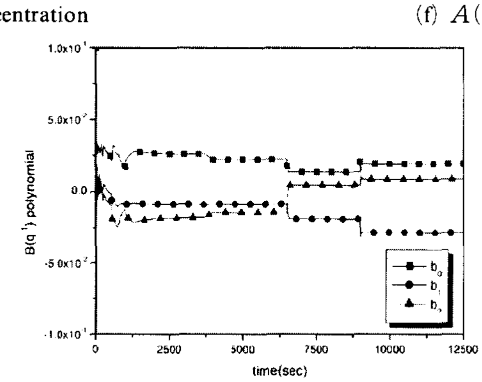
(d) Axial shape index (ASI)



(e) Boron concentration



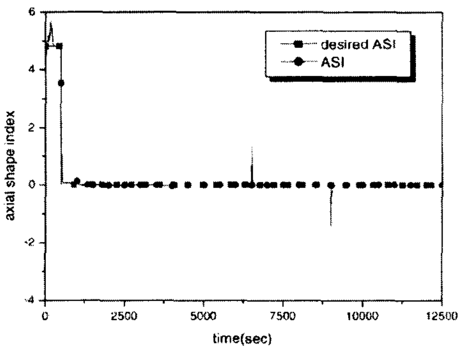
(f) $A(q^{-1})$ polynomial



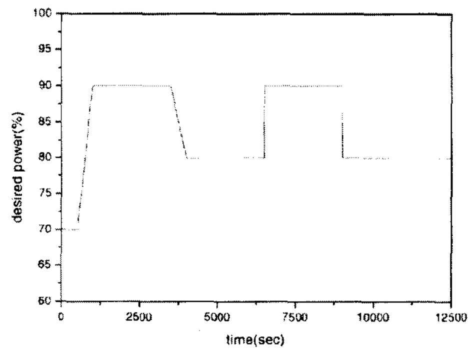
(g) $B(q^{-1})$ polynomial

Fig. 7 Power level control by a constrained model predictive controller

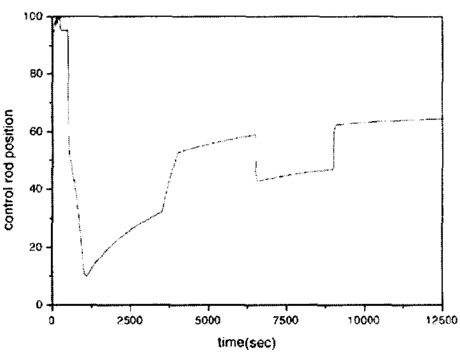
In addition, a conventional proportional-integral (PI) controller was designed to compare the performances for the power level and the power distribution with the proposed model predictive controller. As shown in Fig. 9, the existing PI controller shows performance similar to the proposed model predictive controller. But if nonlinear characteristics are strong because of nuclear fuel burnup and boron concentration change that are not considered in this work, it is expected that the proposed model predictive controller show better performance than the PI controller. Especially, the proposed model predictive control method will be beneficial for the multi-input and multi-output (MIMO) control system to be extended in a further study although not applied in this work.



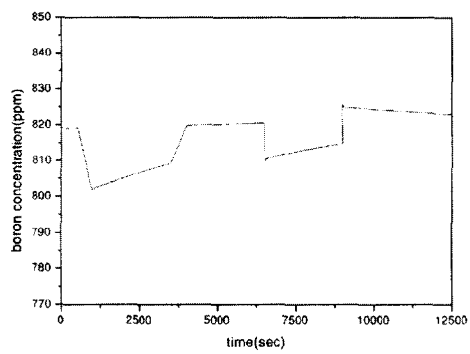
(a) Axial shape index (ASI)



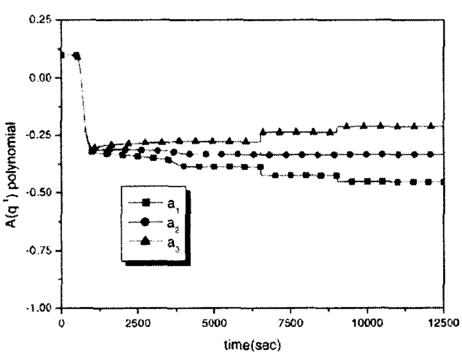
(b) Power level



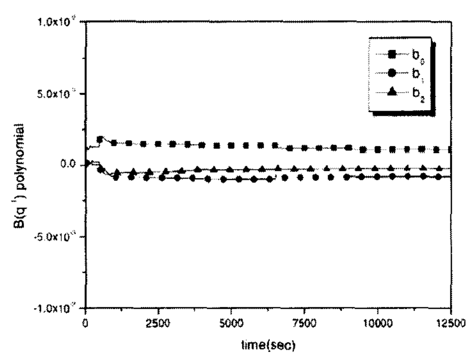
(c) Part-strength control rod position



(d) Boron concentration

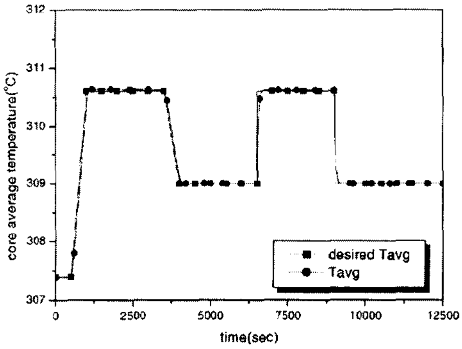


(e) $A(q^{-1})$ polynomial

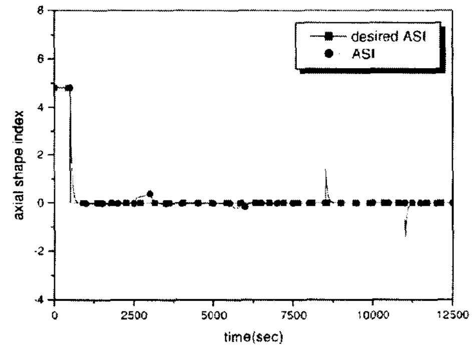


(f) $B(q^{-1})$ polynomial

Fig. 8 Axial shape index (ASI) control by a constrained model predictive controller



(a) Power level control



(b) Axial shape index control

Fig. 9 Power level and axial shape index (ASI) control by existing PI controllers

6. Conclusions

In this work, the model predictive controller has been developed to control the nuclear reactor in pressurized water reactor and the developed controller has been applied to YGN-3 which is modeled by the MASTER code. And a controller design model used for designing the model predictive controller is estimated every time step by applying a parameter estimation algorithm to reflect the time-varying condition. It was known that the proposed controller well follows the desired output (power level or axial shape index) at the 5%/min ramp increase or decrease of a desired load and its 10% step increase or decrease which are design requirements. But these individual controllers for power level and ASI is a little insufficient to reflect real situations, which requires that the two controllers will be integrated. As a further study, the foregoing single input and single output (SISO) control problem will be changed into an MIMO one to reflect real situations.

References

Cho, N. Z. and Grossman, L. M., 1983, "Optimal Control for Xenon Spatial Oscillations in Load Follow of a Nuclear Reactor," *Nuclear Science and Engineering*, Vol. 83, pp. 136~148.

Choi, J. I., Oh, S. Y., Song, I. H., Hah, Y. J., Kuh, J. E. and Lee, U. C., 1992, "Advanced Load Follow Operation Mode for Korean Standardized Nuclear Power Plants," *J. of Korean Nuclear Society*, Vol. 24, No. 2, pp. 183~192.

Clarke, D. W. and Scattolini, R., 1991, "Constrained Receding-Horizon Predictive Control," *IEE Proceedings-D*, Vol. 138, No. 4, pp. 347~354.

Duderstadt, J. J. and Hamilton, L. J., 1976, *Nuclear Reactor Analysis*, John Wiley and Sons, Inc., New York.

Electric Power Research Institute (EPRI), 1992, "Integrated Instrumentation and Upgrade Plan," Rev. 3. *EPRI NP*-7343.

Garcia, C. E., Prett, D. M. and Morari, M., 1989, "Model Predictive Control : Theory and Practice - A Survey," *Automatica*, Vol. 25, No. 3, pp. 335~348.

Gondal, I. A. and Axford, R. A., 1986, "Optimal Xenon Control in Heterogeneous Reactor," *IEEE Trans. Nucl. Sci.*, Vol. NS-33, p. 1722.

Kim, Y. H., Park, M. G., Kim, Y. B. and Choi, Y. S., 1999, "Load Follow Performance of KNGR Using an Extended Mode-K Control Logic," *Proc. of KNS Spring Mtg.*, pp. 1~14, Pohang, Korea.

Kothare, M. V., Balakrishnan, V. and Morari, M., 1996, "Robust Constrained Model Predictive Control Using Linear Matrix Inequality," *Automatica*, Vol. 32, No. 10, pp. 1361~1379.

Kwon, W. H. and Pearson, A. E., 1977, "A Modified Quadratic Cost Problem and Feedback Stabilization of a Linear System," *IEEE Transactions on Automatic Control*, Vol. 22, No. 5, pp. 838~842.

Lee, J. W., Kwon, W. H. and Lee, J. H., 1997, "Receding Horizon Tracking Control for Time-Varying Discrete Linear Systems," *International Journal of Control*, Vol. 68, No. 2, pp. 385~399.

Lee, J. W., Kwon, W. H. and Choi, J., 1998, "On Stability of Constrained Receding Horizon Control with Finite Terminal Weighting Matrix," *Automatica*, Vol. 34, No. 12, pp. 1607~1612.

Lin, C., Chang, J. -R. and Jenc, S. C., 1989, "Robust Control of a Boiling Water Reactor," *Nuclear Science and Engineering*, Vol. 102, pp. 283~294.

MathWorks, 1999, *MATLAB 5.3 (Release 11)*, The MathWorks, Natick, Massachusetts.

Niar, P. P. and Gopal, M., 1987, "Sensitivity-Reduced Design for a Nuclear Pressurized Water Reactor," *IEEE Transactions on Nuclear Science*, Vol. NS-34, pp. 1834~1842.

Park, M. G. and Cho, N. Z., 1993, "Time-Optimal Control of Nuclear Reactor Power with Adaptive Proportional-Integral Feedforward Gains," *IEEE Transactions on Nuclear Science*, Vol. 40, No. 3, pp. 266~270.

Park, Y. H. and Cho, N. Z., 1992, "A Compensator Design Controlling Neutron Flux Distribution via Observer Theory," *Ann. Nucl. Energy*, Vol. 19, No. 9, pp. 513~525.

Richalet, J., Rault, A., Testud, J. L. and Papon, J., 1978, "Model Predictive Heuristic Control: Applications to Industrial Processes," *Automatica*, Vol. 14, pp. 413~428.

Yoon, M. H. and No, H. C., 1985, "Direct Numerical Technique of Mathematical Programming for Optimal Control of Xenon Oscillation in Load Following Operation," *Nucl. Sci. Eng.*, Vol. 90, pp. 203~220.

Winokur, M. and Tepper, L., 1984, "Extension of Load Power Capability of a PWR Reactor by Optimal Control," *IEEE Trans. Nucl. Sci.*, Vol. NS-31, p. 932.